Scaling Laws and Intermittency in Highly Compressible Turbulence

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Established characteristics of high Mach number turbulence

With higher Mach numbers:

- Velocity power spectra gets steeper, reaching Burgers slope -2 asymptotically [Biskamp, 2003]
- density power spectra gets shallower, approaching a slope of -1 [Kritsuk et al., 2006a]
- the density PDF in isothermal flows is well represented by a lognormal distribution [Biskamp, 2003]
- Singular structure dimensionality increases from $D_s = 1$ (subsonic) to $D_s = 2$ (supersonic) [Padoan et al., 2004]
- the mass dimension decreases from $D_m = 3$ (weakly compressible) to $D_m \sim 2.5$ (highly compressible) [Kritsuk et al., 2007]

Numeric turbulence (isothermal, Ma=6, 2048^3?)

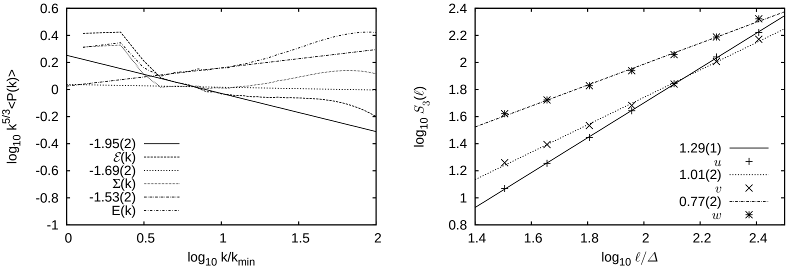

Figure: Time average compensated power spectra (*left*) and third-order transverse(!) structure functions (*right*) for velocity u and mass-weighted velocities $v \equiv \rho^{1/3}u$ and $w \equiv \rho^{1/2}u$. The statistics of v clearly demonstrate a K41-like scaling. Notice strong bottleneck contamination in the spectra at high wavenumbers. [Kritsuk et al., 2007]

Incompressible turbulence

$$\langle\epsilon\rangle = \frac{\partial}{\partial t} e_{kin}(l) = \frac{u^2(l)}{l/u(l)} = \text{const. for } \eta_K \ll l \ll L$$

$$u^p \sim \ell^{\,p/3}$$

$$S_p(\ell) \equiv \langle |u(r+\ell) - u(r)|^p \rangle \sim \ell^{\,p/3}.$$

[Kolmogorov, 1941a], [Kolmogorov, 1941b]

Compressible turbulence

$$\langle\rho\epsilon\rangle = \frac{\partial}{\partial t} \rho e_{kin}(l) = \frac{\rho u^2(l)}{l/u(l)} = \text{const. for } \eta_K \ll l \ll L$$

$$v^p \equiv (\rho^{1/3} u)^p \sim \ell^{\,p/3}$$

$$\mathcal{S}_p(\ell) \equiv \langle |v(r+\ell) - v(r)|^p \rangle \sim \ell^{\,p/3}.$$

[von Weizsäcker, 1951], [Lighthill, 1955], [Fleck, 1996]

Fleck scaling relations

$$u \sim \ell^{1/3+\alpha}, \quad \mathcal{E}(k) \sim k^{-5/3-2\alpha},$$
$$\rho \sim \ell^{-3\alpha}, \quad M(\ell) \sim \ell^{D_m} \sim \ell^{3-3\alpha},$$

The numerical simulations of [Kritsuk et al., 2007] show

$$u \sim \ell^{0.54} \Longrightarrow \alpha = 0.54 - 1/3 = 0.21$$
$$\Longrightarrow D_m = 3 - 3\alpha = 2.38$$

Also one could derive (but it is not done in the actual paper):

$$\mathcal{E}(k) \sim k^{-5/3-2\alpha} \sim k^{-2.09}$$
$$\rho \sim \ell^{-3\alpha} \sim \ell^{-0.62}$$

[Kritsuk et al., 2007] measured $D_m = 2.39$ and $\mathcal{E}(k) \sim k^{-1.95}$.

Numeric turbulence (isothermal, Ma=6, 2048^3?)

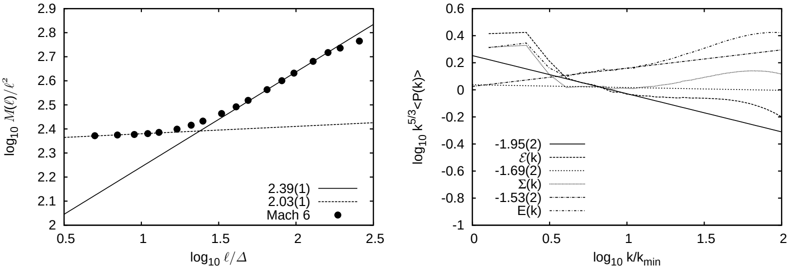

Figure: Left: Gas mass $M(\ell)$ as a function of the box size ℓ (*left*). The mass dimension D_m is defined as the log-log slope of $M(\ell)$. Right: Time average compensated power spectra [Kritsuk et al., 2007]

More about fractal dimensions

- Interface dimension:

$$D_i = 2 + \zeta_1$$

[Meneveau & Sreenivasan, 1990]

Simulations of show $\zeta_1 = 0.54$. This leads to

$$D_i = 2.54 \neq D_m \text{ (!?)}$$

[Kritsuk et al., 2007]

- Fractal dimension of molecular clouds:

$$D = 2.3 \pm 0.3$$

[Elmegreen & Falgarone, 1996], [Elmegreen & Elmegreen, 2001]

She-Leveque & Bolyrev model of turbulence

$$\zeta_p/\zeta_3 = \gamma p + C(1 - \beta^p)$$

[She & Leveque, 1994]

For $p = 3$ it can be solved for C:

$$C \equiv 3 - D_{s,v} = (1 - 3\gamma)/(1 - \beta^3)$$

Best-fit of [Kritsuk et al., 2007] is

$$\beta^3 = 1/3 \quad \text{(a measurement for intermittency)}$$
$$\gamma = 0 \quad \text{(a measure of singularity of structures)}$$
$$\implies D_{s,v} = 1.5 \quad \text{(fractal dimension of dissipative structures)}$$

This corresponds to a hybrid between

- Boldyrev ($\beta^3 = 1/3$, $\gamma = 1/9$) [Boldyrev, 2002] and
- Burgers' model ($\beta^3 = 0$, $\gamma = 0$) [Bec & Khanin, 2007].

Conclusion & further outlook

- The mass-weighted velocity $v \equiv \rho^{1/3} u$ should replace the velocity in intermittency models for compressible flows at high Mach numbers.
- The sensitivity of the results to the forcing needs to be investigated.
- Confusion about fractal dimensions needs to be clarified.
- What about longitudinal structure functions?
- What about compressible, adiabatic turbulence?

Literature

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