Turbulence and first stars in the universe

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Why first stars?

- Pop. III stars had important consequences on galaxy formation.
- UV photons from first stars reionized the universe.
- Enrichment of the IGM with heavy elements is strongly dependent on the mass of the first stars.

Most important questions:

- How big was the mass of the first stars?
- What was the IMF (Initial mass function) of the first stars?

Understanding the evolution of first stars

Good things:

- Initial conditions are simple.
- Chemistry of primordial clouds is simple.
- Magnetic fields are unlikely to be significant.
- Complications by the feedback of stars on the formation of other stars are avoided.

Bad things:

- Many details of the non-linear evolution of stars and galaxies are not understood analytically.
- We have no observational data to guide us.

$\Rrightarrow$ Computer Simulations $\Lleftarrow$

Important thing we have to simulate:

- Gravity on a comoving background mainly due to collisonless cold dark matter.
- Selfgravitating baryonic fluid in the regime of high Reynolds and high Mach numbers.
- Gravitational collapse of the baryonic fluid due to cooling.

Gravitational collapse

- In equilibrium gravity of a gas cloud is balanced by the pressure gradient.
- If there is only thermal pressure and if the mass of the gas cloud is greater than

$$M_J \sim \sqrt{\frac{c_s^6}{\rho}} \sim \sqrt{\frac{p^3}{\rho^4}} \sim \sqrt{\frac{T^3}{\rho}}$$

  the cloud collapses (Jeans criterium).
- But: Adiabatic collapse $\Rightarrow$ increasing thermal pressure $\Rightarrow$ increasing temperature $\Rightarrow$ increasing Jeans mass $\Rightarrow$ collapse ends! $\Longrightarrow$ Gravitational collapse and star formation only possible if the gas cloud can cool non-adiabatically (so entropy rises).
- But: Pressure $=$ thermal pressure $+$ rotational energy density $+$ magnetic pressure $+$ turbulent pressure.

What is turbulent pressure?

Turbulence = velocity fluctuations on all scales.

- Because we cannot resolve all scales in a cosmological simulation (even with AMR), there are always unresolved velocity fluctuations.
- Associated with unresolved velocity fluctuations is unresolved kinetic energy[1] .

Total kinetic energy − resolved kinetic energy = turbulent energy.

$\Longrightarrow$ Turbulent energy density = turbulent pressure.

[1] unresolved kinetic energy ≠ internal energy, because it can be converted by 100% to resolved kinetic energy.

Expressing turbulent energy with Germano filtering

With some arbitrary filter operation with cutoff length Δ

$$\langle f(x)\rangle = \int G_\Delta(x' - x) f(x) dx'$$

we can express the turbulent energy like

$$\begin{array}{ccccc} \tau(v_i, v_i) & = & \langle v_i v_i \rangle & - & \langle v_i \rangle\langle v_i \rangle \\ \text{turbulent energy} & = & \text{total kinetic energy} & - & \text{resolved kinetic energy} \end{array}$$

For a tensor product $v_i v_j$ we get generalized central moments

$$\tau(v_i, v_j) = \langle v_i v_j \rangle - \langle v_i \rangle\langle v_j \rangle$$

Turbulent energy is the trace of $\tau(v_i, v_j)$.

Fluid dynamic balance equations

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial r_j}(v_j \rho) = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial r_j}(v_j \rho v_i) = -\frac{\partial}{\partial r_i}p + \frac{\partial}{\partial r_j}\sigma'_{ij} + \rho g_i$$

Filtered fluid dynamic balance equations

$$\frac{\partial}{\partial t}\langle\rho\rangle + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle = 0$$

$$\frac{\partial}{\partial t}\langle\rho\rangle\hat{v}_i + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle\hat{v}_i = -\frac{\partial}{\partial r_i}\langle p\rangle + \frac{\partial}{\partial r_j}\langle\sigma'_{ij}\rangle + \langle\rho\rangle\hat{g}_i - \frac{\partial}{\partial r_j}\hat{\tau}(v_i, v_j)$$

Filtered fluid dynamic balance equations

$$\frac{\partial}{\partial t}\langle\rho\rangle + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle = 0$$

$$\frac{\partial}{\partial t}\langle\rho\rangle\hat{v}_i + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle\hat{v}_i = -\frac{\partial}{\partial r_i}\langle p\rangle + \langle\rho\rangle\hat{g}_i - \frac{\partial}{\partial r_j}\hat{\tau}_{ij}$$

with an eddy-viscosity model for

$$\hat{\tau}_{ij} = \hat{\tau}^*_{ij} + \frac{1}{3}\delta_{ij}\hat{\tau}_{ii} = 2\eta_t S^*_{ij} + \frac{2}{3}\delta_{ij}e_t$$

where

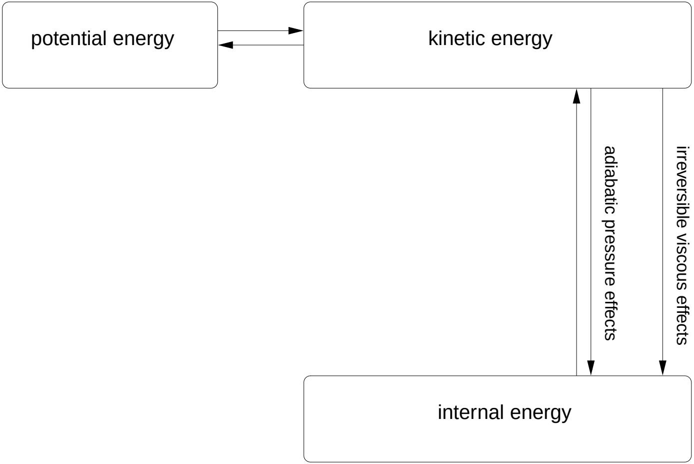
$$\eta_t \sim \sqrt{e_t}\ ; \quad S^*_{ij} = \frac{1}{2}\left(\frac{\partial\hat{v}_i}{\partial x_j} + \frac{\partial\hat{v}_j}{\partial x_i}\right) - \frac{1}{3}\delta_{ij}\frac{\partial\hat{v}_k}{\partial x_k}$$

Filtered fluid dynamic balance equations

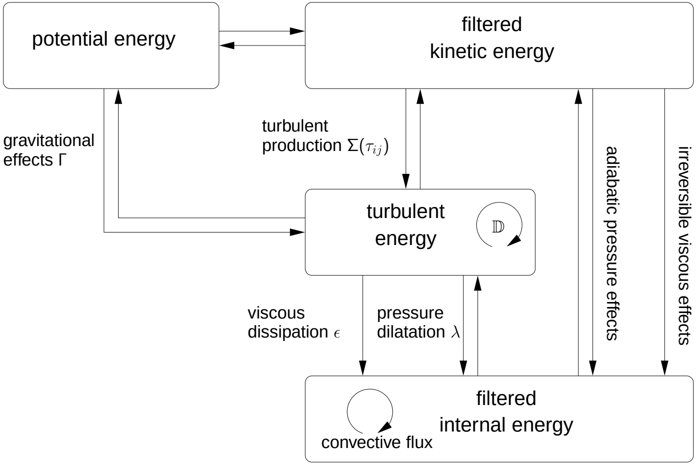
$$\frac{\partial}{\partial t}\langle\rho\rangle + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle = 0$$

$$\frac{\partial}{\partial t}\langle\rho\rangle\hat{v}_i + \frac{\partial}{\partial r_j}\hat{v}_j\langle\rho\rangle\hat{v}_i = -\frac{\partial}{\partial r_i}\left(\langle p\rangle + \frac{2}{3}e_t\right) + \langle\rho\rangle\hat{g}_i - 2\eta_t\frac{\partial}{\partial r_j}S^*_{ij}$$

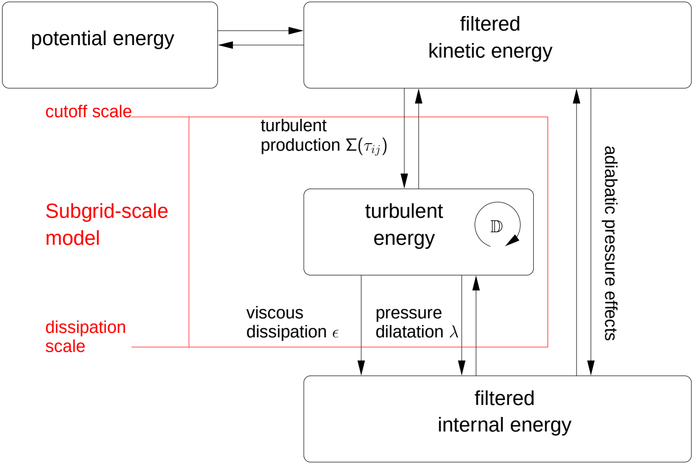
Energy equation scheme



Energy equation scheme



Energy equation scheme



SGS model for cosmology and first star simulations

- Basic assumption for Subgrid-scale models is isotropic turbulence.
- But: Cosmological Simulations show a lot of anisotropic flow features on large scales $\Rightarrow$ AMR!
- AMR $\Rightarrow$ spatially and temporarily varying grid scale.

$\Longrightarrow$ Localized subgrid-scale model (Schmidt et. al., 2006)

SGS + AMR = FEARLESS[2]

[2] Fluid mEchanics with Adaptively Refined Large- Eddy SimulationS

Turbulent dissipation on a varying grid 1

$$\epsilon_r(\Delta) = C_{\epsilon,r} \frac{(2e_{t,r})^{3/2}}{\left(\frac{\Delta}{r}\right)}$$

Δ = root grid length,
r = refinement factor

Basic Assumptions

Locally conserved for different refinement factors are

- the dissipation ϵ (and C_ϵ):

$$\epsilon_{r_1}(\Delta) = \epsilon_{r_2}(\Delta)$$

- the sum of turbulent energy and kinetic energy:

$$\frac{1}{2}v_{r_1}^2 + e_{t,r_1} = \frac{1}{2}v_{r_2}^2 + e_{t,r_2}$$

Turbulent dissipation on a varying grid 2

From the first assumption we get

$$\frac{e_{t,r_1}}{e_{t,r_2}} = \left(\frac{r_2}{r_1}\right)^{2/3}. \tag{1}$$

Using this in the second assumption we get

$$v_{r_2}^2 = v_{r_1}^2 \left[2\left(\frac{e_{t,r_1}}{v_{r_1}^2}\right)\left(1 - \left(\frac{r_2}{r_1}\right)^{-2/3}\right)\right] \tag{2}$$

Transfer of turbulent energy during grid refinement

1. Interpolate values from coarse to fine grid using standard interpolation scheme from ENZO
2. Correct interpolated values (here $r_1 = 1, r_2 = r$)

kinetic energy: $$\frac{1}{2}v_r^2 = \frac{1}{2}v_1^2 + e_{t,1}\left(1 - r^{-2/3}\right)$$

velocity components: $$v_{i,r} = v_{i,1}\sqrt{1 + 2\frac{e_{t,1}}{v_1^2}\left(1 - r^{-2/3}\right)}$$

turbulent energy: $$e_{t,r} = e_{t,1} - e_{t,1}\left(1 - r^{-2/3}\right)$$

What do we know about the interpolation?

- **InterpolationMethod** (external) - There should be a whole section devoted to the interpolation method, which is used to generate new sub-grids and to fill in the boundary zones of old sub-grids, but a brief summary must suffice. The possible values of this integer flag are shown in the table below. The names specify (in at least a rough sense) the order of the leading error term for a spatial Taylor expansion, as well as a letter for possible variants within that order. The basic problem is that you would like your interpolation method to be: multi-dimensional, accurate, monotonic and conservative. There doesn't appear to be much literature on this, so I've had to experiment. The first one (ThirdOrderA) is time-consuming and probably not all that accurate. The second one (SecondOrderA) is the workhorse: it's only problem is that it is not always symmetric. The next one (SecondOrderB) is a failed experiment, and SecondOrderC is not conservative. FirstOrderA is everything except for accurate. If HydroMethod = 2 (ZEUS), this flag is ignored, and the code automatically uses SecondOrderC for velocities and FirstOrderA for cell-centered quantities. Default: 1

0 - ThirdOrderA	3 - SecondOrderC
1 - SecondOrderA	4 - FirstOrderA
2 - SecondOrderB	

Is there a interpolation that is locally conservative and leaves the spectrum invariant when refining the grid?

State of FEARLESS

Done:

- Reformulated Schmidt SGS in comoving coordinates.
- Integrated Schmidt SGS into cosmological code ENZO without using AMR.
- Implemented correction of turbulent energy when generating finer grids to make Schmidt SGS work consistently with AMR.

To-Do-List:

- Add Schmidt SGS with dynamic coefficients.
- Do first star simulations in analogy to Abel, et al. 2001 (Initial conditions for turbulent energy?)