Mini-Workshop on Lorentz Violation

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Introduction

Einstein's theory of special relativity has been spectacularly successful in accounting for a vast number of physical phenomena, and the consequential Lorentz symmetry has proven extremely successful as a guiding principle for formulating new laws of physics. In light of very strong constraints which have been derived on low energy violations of Lorentz symmetry and the fact that - until today - no conclusive evidence for its violation even for high boost factors has been found, it seems tempting to assume it to be a unbroken symmetry of nature, valid for boosts at least up to the Planck scale.
Over the last decades, a tremendous amount of work has been dedicated to tests of Lorentz symmetry and Lorentz violating theories [1, 2, 3, 4, 5]. Due to the non-compact nature of the Lorentz group the hypothesis of it being an exact symmetry in nature can never be confirmed and so there always remains the possibility for it to be violated at some scale. Indeed, modern physical theories suggest the existence of Lorentz violation (LV) observable generically at energies near the Planck scale. Among the most prominent approaches are string theory [6], models of quantum gravity [7] and non-commutative (NC) spacetime geometries [8], with no attempt to give a complete list. Moreover, recent astrophysical observations reported results which might be explained by broken Lorentz symmetry [9], even though it needs to be stated that those results are at best marginally conclusive and the observed effects can also be explained employing exotic but standard Lorentz invariant physics.

I. MOTIVATION

With the unique possibility to combine the knowledge of specialists in the field of particle physics - especially of NC theories - with the expertise of astrophysicists we were taking on our own approach toward the challenging topic of LV. In the framework of a two day mini-workshop we introduced ourselves to different aspects of LV driven by the unique focus and experience of each member of the group. We started out with an introduction to the most general formulation of LV effects by means of effective interactions and an effective parametrization of dispersion relations, and then discussed the more specific case of LV from NC theories. Furthermore, a brief introduction to the derivation of constraints on LV from astrophysical observations was presented, followed by a discussion of objectives and strategies.
From this basis we went on to consider several specific problems. Among them were the observations of Ultra High Energy (UHE) Cosmic Rays with the experiments HiRes and Fly's Eye which are presenting opposing results to the AGASA experiment, illustrating the current problems with the GZK cutoff [9]. On the theory side, some work was dedicated to the emergence of LV terms in dispersion relations and the question of specific scenarios (i.e. relativistic AGN jets, Gamma-ray bursts (GRB)) in which one should have the best opportunity to obtain good observational results.
To both gain a more qualitative and quantitative insight to LV we decided to focus on two aspects: firstly, to consider astrophysical constraints in the framework of an effective theory as presented in section (II), and secondly to work out a dispersion relation for photons from NC spacetime in a specific scenario as outlined in section (III).

II. SECTION 2

Dispersion relations inferring Lorentz invariance breaking

One of the simplest kinematic frameworks for LV is to propose modified dispersion relations for particles , while keeping the usual energy-momentum conservation laws.

An effective field theory for breaking the Lorentz invariance is introduced, that considers expansions in integer powers of the momentum:

$$E^2 = p^2 + m^2 + \sum_{n=1}^{\infty} a_n p^n \,, \quad (1)$$

with $p = |\vec{p}|$. If Lorentz invariance violation is associated with quantum gravity, deviations from ordinary Lorentz invariance should appear at the Planck scale $M_{\rm Pl} = \sqrt{\hbar^5/G} = 1.22 \cdot 10^{19} GeV$. Lets write

$$a_n = \frac{\eta_n}{M_{\rm Pl}^{n-2}} \,. \quad (2)$$

η_n is a dimensionless factor, which can in principle be different for different particles. Considering $n = 3, 4$ ($n < 3$ ruled out by previous terrestial experiments. At high energies the lowest nonzero term with $n \geq 3$ will dominate). The dispersion relation then gets:

$$E^2 = p^2 + m^2 + \eta_a \frac{p^n}{M_{\rm Pl}^{n-2}} \quad (3)$$

where a denotes different particle types. Such corrections might only become important at the Planck scale, but there are two exclusions:
1. Particles that propagate over cosmological distances can show differences in their propagation speed.
2. Energy thresholds for particle reactions can be shifted or even forbidden processes can be allowed. If the p^n-term is comparable to the m^2-term in equation 3, threshold reactions can be significantly shifted, because they are determined by the particle masses. So a threshold shift should appear at

$$p_{\rm dev} \approx \left(\frac{m^2 M_{\rm Pl}^{n-2}}{\eta} \right)^{1/n} \,. \quad (4)$$

Assuming $\eta \approx 1$ the typical scales for the thresholds for different particles are given in table I.

n	$p_{\rm dev}$ for ν_e	$p_{\rm dev}$ for e^-	$p_{\rm dev}$ for p^+
3	≈1 GeV	≈10 TeV	≈1 PeV
4	≈100 TeV	≈100 PeV	≈3 EeV

Table I: Typical scales for energy thresholds for different particles where threshold reactions can be shifted by Lorentz invariance breaking

In the following we consider three processes involving Lorentz invariance breaking.

1. Time of flight measurements,
2. Threshold creation for
   - Vacuum Cherenkov effect,
   - Photon decay,
3. Shift of GZK cutoff.

This list does not attempt to be complete. By these tests one can estimate constraints for the correction factors η_n, depending on the different particles and reactions taking place.
The dispersion relation (equation 1) is only one approach for introducing Lorentz invariance violation. [1]
(Other phenomena like non-commutativity lead to a dispersion relation which is independent of the energy of the particle, at least in the case of photons. Constraining Lorentz invariance violation by possible non-commutative space-time effects will be discussed in section III.)

The time of flight approach

A modified dispersion relation, like introduced in the previous section would lead to an energy dependent speed of light. The idea of the time of flight (TOF) approach is to detect a shift in the arrival time of photons with different energies, produced simultaneous in a distant object, where the distance gains the usually Planck suppressed effect [10]. In the following we use the dispersion relation for $n = 3$, as modifications in higer orders are far below the sensitivity of current or planned experiments. The modified group velocity is then:

$$v = \frac{\partial E}{\partial p} \quad (5)$$

$$v \approx 1 - \eta_\gamma \frac{p}{M} \quad (6)$$

For the time difference one gets

$$\Delta t = \eta_\gamma \frac{p}{M} D \quad (7)$$

where D is the distance multiplied by $(1 + z)$ to correct the energy for the redshift.
In the recent years, several measurements on different objects in various energy bands leading to constraints up to the order of 100 for η (see table II), where the best constraint comes from a short flare of the Active Galactic Nucleus (AGN) Mrk 421, detected in the TeV band by the Whipple Imaging Air Cherenkov telescope.

Object	ΔE	Δt	D[light sec]	η	ref.
pulsars	2 GeV	0.35 ms	2.0×10^{11}	10^4	[11]
GRB	270 keV	0.35 ms	10^{17}	10^3	[12]
AGN	1 TeV	280s	10^{16}	10^2	[13]
GRB	100 GeV	0.1 ms	10^{17}	10^{-4}	this work

Table II: List of different astronomical objets, used for LV testing. The values for GRB observation from the last line is anticipated observations with the upcoming GLAST mission (see text).

There is still room for improvements with current or planned experiments, although the distance for TeV-observations is limited by absorption of TeV photons in low energy Meta galactic radiation fields ([14], [15]). Depending on the energy density of the target photon field one gets an energy dependent mean free path length, leading to an energy and redshift dependent cut off energy (the cut off energy is defined as the energy where the optical depth is one).
Figure 1 shows the energy difference versus the arrival time difference for a modified dispersion relation in order of the Planck scale ($\eta = 1$) for different redshifts. The shaded area shows the part where the gamma-ray emission is significantly attenuated by absorption, limiting TeV observations to nearby ($z < 0.1$) objects.
Below $\sim$ 50GeV the universe is optically thin. This leads to the best possibility for the GLAST satellite (launch planned for 2007) possibly improving the current constraints up to six orders of magnitude. For this assumption, the results from BATSE on GRB was used, just by scaling to the GLAST energy range. This may be too optimistic, because the photon fluxes at these energies are much lower than in the hard X-ray regime, making a time resolution below ms hardly feasable. Therefore it can be seen as an upper limit for the possibilities of the near future. At energies above 50 TeV the universe becomes optically thin again. Although there will be future experiments, sensitive for observations in the 100 TeV range currently under discussion, the photons fluxes are expected to be so low, that resolvable time structure would be above days or even weeks yielding much worse constraints on LV. The advantage of the TOF method is

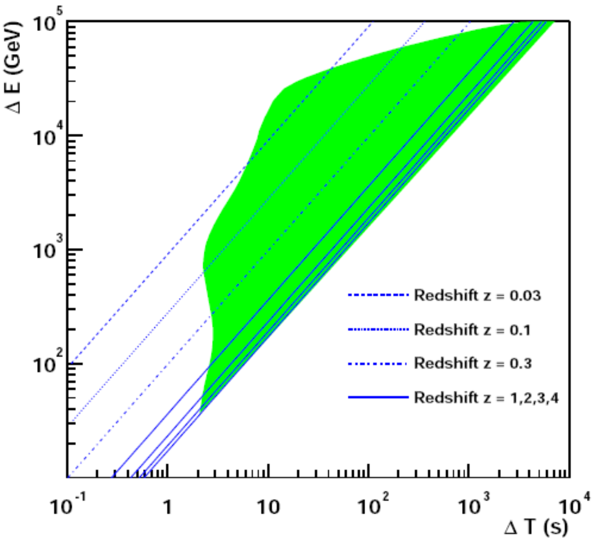

Figure 1: Energy difference vs. arrival time difference for $\eta = 1$ for different redshifts. The shaded area indicates the region with significant attenuation of gamma-rays [16].

the very general approach, not depending on a special model. Current constrains will be improved by some orders of magnitude by future experiments, although there seems to be a limitation based on the combination of high energies, limited distances and low photon fluxes (being responsible for a poor time resolution).

Photon-Electron processes

The interaction vertex in quantum electrodynamics (QED) couples one photon with two leptons. When we assume for photons and leptons the following dispersion relations (for simplicity we adopt all units with $M = 1$, see Eqn. 3):

$$\omega_k^2 = k^2 + \xi k^n, \tag{8}$$
$$E_p^2 = m^2 + p^2 + \eta p^n, \tag{9}$$

and denote the photon 4-momentum by $k_4 = (\omega_k, \vec{k})$ and the two lepton 4-momenta by $p_4 = (E_p, \vec{p})$ and $q_4 = (E_q, \vec{q})$ we receive:

$$\xi k^n + \eta p^n - \eta q^n = 2\left(E_p\omega_k - \vec{p}\vec{k}\right), \tag{10}$$

where the r.h.s. is always positive. In the Lorentz invariant case the parameters ξ and η are zero, so that this equation can't be solved and all processes of the single vertex are forbidden. If these parameters are non-zero, there can exist a solution and so these processes can be allowed.
We now consider two of these interactions to derive constraints on the parameters ξ and η: The vacuum Cherenkov effect ($e^- \rightarrow \gamma\, e^-$) and the spontaneous photon-decay ($\gamma \rightarrow e^+\, e^-$).

A. Vacuum Cherenkov effect

The vacuum Cherenkov effect is a spontaneous emission of a photon by a charged particle ($0 < E_\gamma < E_{\rm par}$). These effect occurs if the particle moves faster than the slowest possible radiated photon.
In the case of $\eta > 0$, the maximal attainable speed for the particle $c_{\rm par}^{\rm max}$ is faster than c. This means, that the particle can always be faster than a zero energy photon with

$$c_{\gamma_0} = c \cdot \lim_{k\to 0}\frac{\partial\omega}{\partial k} = c \cdot \lim_{k\to 0}\frac{1}{2}\frac{2k + n\xi k^{n-1}}{\sqrt{k^2+\xi k^n}} = c \tag{11}$$

independent of ξ.
In the case of $\eta < 0$ (c_{par} decreases with energy) you need a photon with $c_\gamma < c_{par} < c$. This is only possible when $\xi < \eta$.
Due to the radiation of photons such an electron loose energy. The observation of high energetic electrons allows

to derive constraints on η and ξ [6]. In the case of $\eta < 0$ we receive for $n = 3$:

$$\eta < \frac{m^2}{2\, p_{max}^3}. \tag{12}$$

From the observation of 50 TeV photons in the Crab Nebula [17] one can conclude the existens of 50 TeV electrons due to the inverse Compton scattering of these e^- with those photons. This leads to a constraint on η of $\eta < 1.2 \times 10^{-2}$ (in the case of $\eta > 0$).

B. Photon decay

The decay of photons into positrons and electrons ($\gamma \to e^+ e^-$) should be a very rapid spontaneous decay process. Due to the observation of Gamma rays from the Crab Nebula on earth with an energy up to $\sim$ 50 TeV [17] we can reason that these rapid decay doesn't occur on energies below 50 TeV. For the constraints on ξ and η these means [6] for $n = 3$:

$$\xi < \frac{\eta}{2} + 0.08 \qquad \text{for } \xi \geq 0, \tag{13}$$

$$\xi < \eta + \sqrt{-0.16\,\eta} \qquad \text{for } \eta < \xi < 0. \tag{14}$$

Shift in GZK cut off

As the energy of a proton increases, the pion production reaction can happen with low energy photons of the Cosmic Microwave Background (CMB). This leads to an energy dependent mean free path length of the particles, resulting in a cutoff at $\approx 10^{20}$ eV, the so-called Greisen-Kuzmin-Zatsepin (GZK) cut off.

$$E_{\rm GZK} = \frac{m_p m_\pi}{2E_\gamma} = 3 * 10^{20} eV \left(\frac{2.7K}{E_\gamma}\right) \tag{15}$$

Thus in Lorentz invariant world, the mean free path length of a particle of energy 5.10^{19} eV is 50 Mpc i.e. particle over this energy are readily absorbed due to pion production reaction. But most of the sources of particle of ultra high energy are outside 50 Mpc [18]. So, one expects no trace of particles of energy above 10^{20} in earth.

From the experimental point of view AGASA has found a few particles having energy higher than the constraint given by GZK cutoff limit and claimed to be disproving the presence of GZK cutoff or at least for different threshold for GZK cutoff, whereas HiRes is consistent with the GZK effect [18]. So, the questions are

1. How one can get definite proof of non-existence GZK cut off?
2. If GZK cutoff doesn't exist, then find out the reason?

The first question could by answered by observation of a large sample of events at these energies, which is necessary for a final conclusion, since the GZK cutoff is a statistical phenomena. the up-coming AUGER experiment, still under construction, may clarify if the GZK cutoff exists or not [22]. The existence of the GZK cutoff would also yield new limits on Lorentz violation.

For the second question, one explanation can be derived from Lorentz violation. If we do the calculation for GZK cutoff in Lorentz violated world we will get the modified proton dispersion relation as described in equation 3. This changes the GZK threshold when the threshold is compared to or greater than m_p^2 at or around E_{GZK} [10]. LV not only may change the location of GZK cutoff even it may lift off the existence of GZK cutoff. Since in the third term all the parameters are positive numbers the out come will be positive, thus E will be less. Now this E comes as denominator in the equation of GZK cutoff, so, evidently GZK limit will be elevated and we can expect higher energy particles from extra galactic objects out side 50 Mpc. Also this calculation gives us a great opportunity to include n=4 in Lorentz violated dispersion relation (since the energy is "close" to the plank scale) with a better constraint on $O(E^n/M^{(n-2)})$. For the case of n=3 dispersion with $f_p^3 = f_\pi^3$ and if the GZK limit saved then we can get a clear constraint on Lorentz violated dispersion term as $f_p^3 < O(10^{-14})$ [1], which would be the strongest constraint from astrophysical observations.

III. SECTION 3

Phenomenological Signatures of Lorentz Violation in Noncommutative Spacetime

Dispersion Relations for Noncommutative Spacetime

In this section we want to concentrate on noncommutative spacetimes for which Lorentz invariance is broken by the nonvanishing commutator

$$[\hat{x}^\mu, \hat{x}^\nu] = \frac{i\theta^{\mu\nu}}{\Lambda_{\rm NC}^2}, \tag{16}$$

where $\theta^{\mu\nu}$ is a constant, antisymmetric Lorentz tensor

$$\theta = \begin{bmatrix} 0 & \tilde{\theta}^1 & \tilde{\theta}^2 & \tilde{\theta}^3 \\ -\tilde{\theta}^1 & 0 & \theta^3 & -\theta^2 \\ -\tilde{\theta}^2 & -\theta^3 & 0 & \theta^1 \\ -\tilde{\theta}^3 & \theta^2 & -\theta^1 & 0 \end{bmatrix} \tag{17}$$

whose entries are of order 1. The scale Λ_{NC} is

typically near the Planck scale [?]. However, models based on large extra dimensions can impose a scale $\Lambda_{\rm NC} \sim$ TeV [?]. This is of particular interest since it may lead to phenomenological observations. Therefore,

Object	Field Size	Field Strength [Tesla]	$\lvert\vec{\theta}\rvert\lvert\vec{B}\rvert/\Lambda_{\rm NC}{}^2$	Δt $[s]$
Magnetar (10^4 m)	$5\cdot 10^4$ m	10^{11}	$2.0\cdot 10^{-11}$	$3\cdot 10^{-15}$
Star (10^9 m)	$5\cdot 10^9$ m	1.0	$2.0\cdot 10^{-22}$	$3\cdot 10^{-21}$
Galaxy (10^{20} m)	$5\cdot 10^{20}$ m	10^{-10}	$2.0\cdot 10^{-32}$	$3\cdot 10^{-20}$

Table III: Time shift for photons passing a magnetic background field due to NC effects.

we investigate in the following whether astrophysical observations are able to constrain certain class of models with noncommutative spacetimes which are broken at the TeV scale. For this we use the dispersion relations for plane waves in noncommutative QED, one using the Moyal-Weyl star product as a realization of the noncommutative algebra and the other one additionally using Seiberg-Witten maps [19]. In the latter approach, the dispersion relation is expanded up to first order in θ. These two pictures are not equivalent, but for the scenarios considered in this letter, *i.e.* a magnetic background field only and $\tilde{\theta}^1 = \tilde{\theta}^2 = \tilde{\theta}^3 = 0$, the differences for the dispersion relations are negligible. In general:

Moyal-Weyl:

$$\omega \;=\; |\vec{k}|\left(|(1+\frac{1}{2}\vec{\mathcal{B}}\cdot\vec{\theta})\hat{k}-\frac{1}{2}(\hat{k}\cdot\vec{\mathcal{B}})\vec{\theta}|-\frac{1}{2}\hat{k}\cdot(\vec{\mathcal{E}}\times\vec{\theta})\right) \tag{18a}$$

Seiberg-Witten:

$$\omega \;=\; |\vec{k}|+\frac{|\vec{k}|}{\Lambda_{NC}^2}\left[\vec{\mathcal{E}}_T\cdot\vec{\tilde{\theta}}_T+\vec{\mathcal{B}}_T\cdot\vec{\theta}_T+\hat{k}\cdot(\vec{\mathcal{B}}_T\times\vec{\tilde{\theta}}_T-\vec{\mathcal{E}}_T\times\vec{\theta}_T)\right], \tag{18b}$$

where the subscript T denotes the transversal component and $\hat{k} = \vec{k}/|\vec{k}|$ and $\vec{\theta} = (\theta^1,\theta^2,\theta^3)^T$. These relations merge, with a vanishing electric background field, *i.e.* $\vec{\mathcal{E}} = 0$, $\vec{\tilde{\theta}} = 0$, $\vec{\theta} \neq 0$ and $|\vec{\mathcal{B}}||\vec{\theta}| \ll 1$ to a reduced form of the dispersion relations respectively:

$$\omega \approx |\vec{k}| + \frac{|\vec{k}|\,\vec{\mathcal{B}}_T\cdot\vec{\theta}_T}{\Lambda_{NC}^2}, \tag{19}$$

where the magnetic field has units of eV^2. From this equation, the velocity and time delays can be derived:

$$v \;=\; 1+\frac{\vec{\mathcal{B}}_T\cdot\vec{\theta}_T}{\Lambda_{\rm NC}^2}, \tag{20a}$$

$$c\,\Delta t \;=\; d_{field}\,\frac{\vec{\mathcal{B}}\cdot\vec{\theta}/\Lambda_{\rm NC}^2}{1+\vec{\mathcal{B}}\cdot\vec{\theta}/\Lambda_{\rm NC}^2} \approx \frac{d_{field}\,\vec{\mathcal{B}}\cdot\vec{\theta}}{\Lambda_{\rm NC}^2}, \tag{20b}$$

where d_{field} is the path length of the photon in the magnetic field. Here we see, that without a magnetic and electric background field, the dispersion relation for photons remains the same as in a commutative spacetime. Furthermore, there is no photon energy dependence of the dispersion relation. Consequently are time-of-flight experiments inappopriate because of their energy-dependent dispersion. This is in comparison with [19], where also no birefringence were found. Therefore, we suggest the following setup:

Suppose, there exists a strong magnetic field [23] (for example from a star or a cluster of stars) on the path photons emitted at a light source (*e.g.* gamma-ray bursts). Then, analogous to gravitational lensing, the photons experience deflection and/or change in time-of-arrival, compared to the same path without a magnetic background field. Estimations for several examples are shown in Table III.

Higher Order Effects

The second order expansion of the Seiberg-Witten map in NCQED in θ leads to conclusion that

$$\theta^{\alpha\rho}\theta^{\beta\delta}(\partial_\alpha\partial_\beta F_{\mu\nu})(\partial_\rho\partial_\delta F^{\mu\nu}) \tag{21}$$

is the only $\mathcal{O}(F^2)$ term (where F denotes the Field Strength) which modifies the vacuum dispersion rela-

tions, with no static electromagnetic background field. However, this term vanishes due to the antisymmetry of θ after partial integration [Thesis Ana].
Nevertheless, in a review by Mattingly [3], the dimension 4 operators of the form $\theta\theta FF$, can arise from loop effects which could give contributions to vacuum dispersion. Furthermore, effects which are in higher orders of the external magnetic field can be neglected.

Conclusions

We investigated phenomenological consequences of spactime noncommutativity which should broken at the TeV scale in an astrophysical context. For this we considered modified photon dispersion relations as an example for Lorentz violation. We found that even a scale $\Lambda_{\rm NC} \ll \Lambda_{Planck}$ yield no mesurable effects arising in astrophysical observations. Nevertheless, Lorentz violation in noncommutative models may be probed by considering dispersion relations for massive particles [20], and so be within the reach of the next generation of collider experiments [21].

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[22] http://www.auger.org/
[23] The conversion factor used for the magnetic field unit is: $1\,\mathrm{Tesla} \simeq 195.38(eV)^2$